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Quasinormal modes and stability of a minimal traversable wormhole

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Abstract

The minimal wormholes are a special kind of the traversable wormholes. These ones are solutions of Einstein's Field Equations that connect two regions asymptotically flat, whereby an observer could travel between them. In this work, which composes the Master's research of Laura Penedos, we characterized a minimal wormhole in terms of their matter content and perturbation by a massless scalar test particle. This perturbation has been based on a Scattering Problem using a Effective Potential whose incident waves were eliminated. Thus, a Transcendental Equation is produced whose solution is a set of quasinormal frequencies and, from their imaginary part, we are able to analyse stability of this wormhole.

Introduction

Wormholes are solutions of the Einstein's Equations connecting two asymptotically flat regions [1]. For a specifically type of these objects, dubbed traversable wormholes, it is possible to a observer to travel across them, establishing a viable path between these regions [2]. Even though exotic matter - which violates the usual conditions of Energy Conservation - is required to support such kind of object, their inherent geometries might offer some important tools to comprehend more complex phenomena.

A special kind of traversable wormhole is called "minimal wormhole" [3]. Its construction is based on the metric by Kar, Minwalla, Mishra and Sahdev (KMMS) [4], which is expressed below:

$$ds^2 = -dt^2 + dl^2 + R(l)^2 d\Omega^2 \tag{1}$$

Where $R(l) = ({b_0}^n + l^n)^{1/n}$ and $d\Omega = d\theta^2 + (sin\theta)^2 d\phi^2$. The minimal wormhole is reached when we set $n \to \infty$ [3].

We have been working with the minimal wormhole in terms of their matter content, and also it behaviour with a massless scalar test particle perturbation. The main goal of our research is to verify the stability of this object.

Minimal Wormhole

We started with the Energy Momentum Tensor of the metric (1). Adopting that $8\pi G = c^4 = 1$ in Einstein's Equations, so $G_{\mu\nu} = T_{\mu\nu}$, we find the following results, when we take the limit $n \to \infty$:

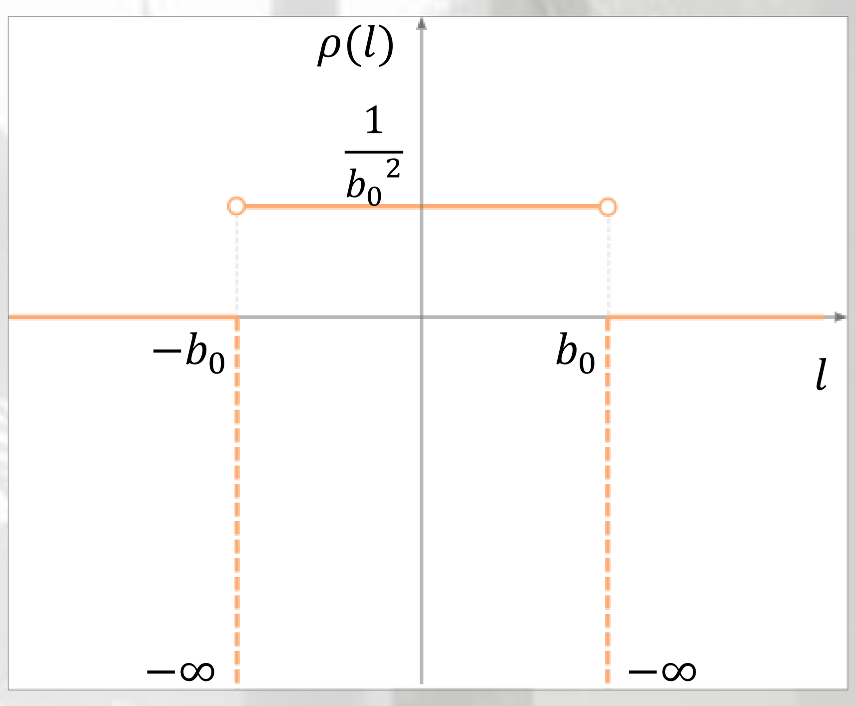

Figure 1: Graph $\rho(l)\ x\ l$, relative to minimal wormhole. No scale.

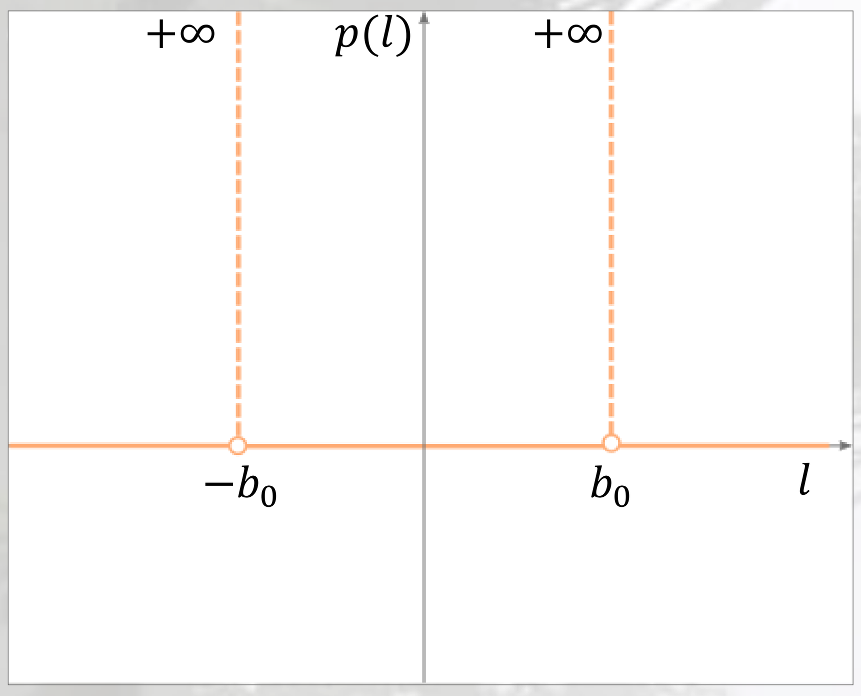

Figure 2: Graph $p(l)\ x\ l$, relative to minimal wormhole. No scale.

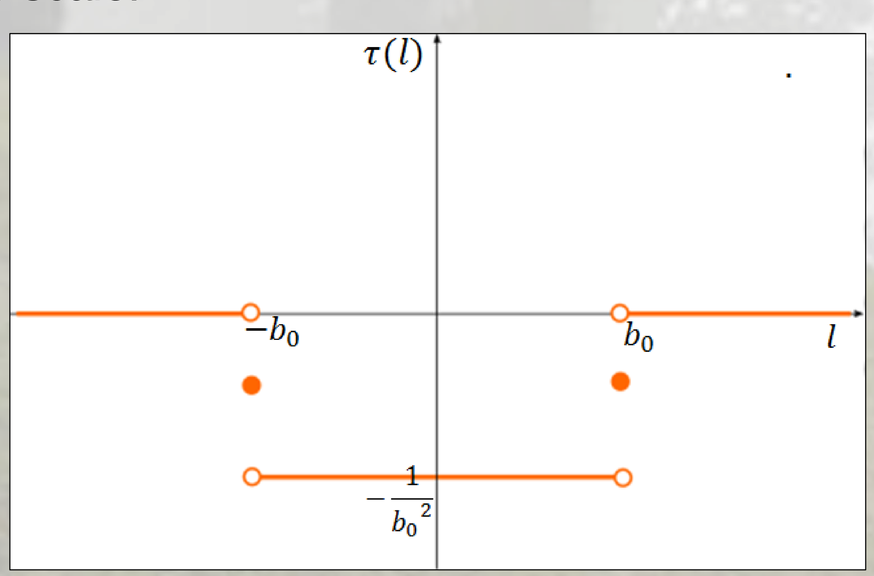

Figure 3: Graph $\tau(l)\ x\ l$, relative to minimal wormhole. No scale.

Secondly, we used the Klein-Gordon Equation for a massless scalar particle in the KMMS wormhole to calculate the Effective Potential. The wavefunction is given by $\varphi(l,\theta,\phi,t) = u(l) Y_{l'm} e^{-i\omega t}$. The Effective Potential appears in an eigenvalue equation, similar to Schrödinger Equation in Quantum Mechanics.

$$\frac{1}{\sqrt{g}} \partial_a \sqrt{g} g^{\alpha\beta} \partial_b \varphi = 0 \Rightarrow \left[-\frac{d^2}{dl^2} + V_{eff}\right] u(l) R(l) = \omega^2 u(l) R(l) \tag{2}$$

For $n \to \infty$, the Effective Potential assumes the form showed in Figure 4.

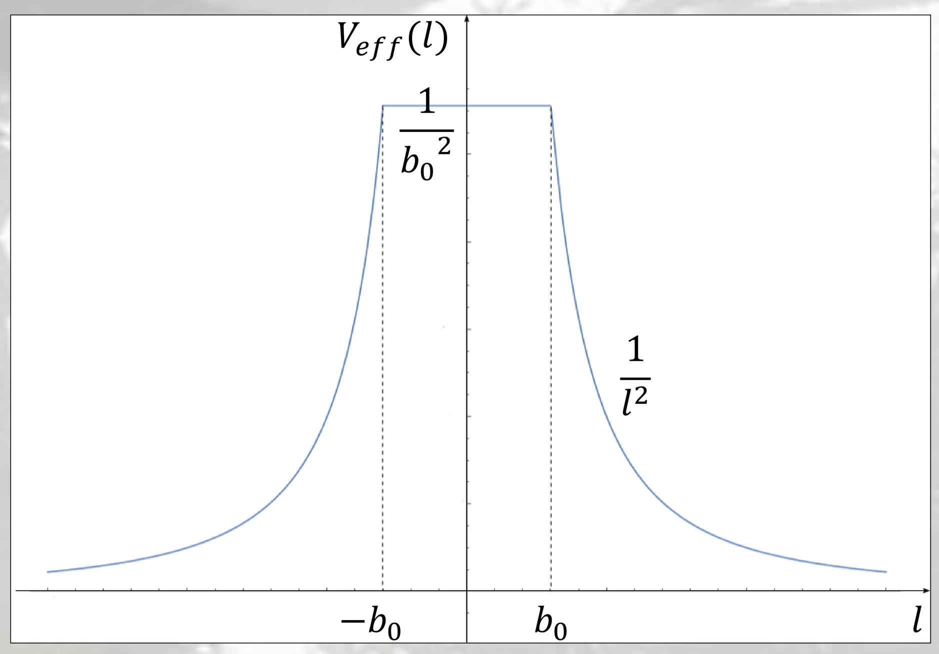

Figure 4: Graph $V_{eff}(l)\ x\ l$, relative to minimal wormhole for massless scalar perturbations. It was adopted $l'(l'+1) = 1$.

Quasinormal frequencies

To calulate the quasinormal frequencies, we started with Effective Potential in Figure 4 and proposed a Scattering Problem where there are no incident waves in outside regions. Thus, we solved the eigenvalue equation (2) for each region, and then applied the boundary conditions (derivative of the wavefunction and the wavefunction itself continuity). The result is a Transcendental Equation whose solutions in ω are the quasinormal frequencies [5]. Our unique condition is $\omega^2 > \frac{1}{{b_0}^2}$.

Region I: $l \le -b_0 \Rightarrow \left[-\frac{d^2}{dl^2} + \frac{1}{l^2}\right] u(l) = \omega^2 u(l) \Rightarrow$ $u_I(l) = A_1 \sqrt{l} H^{(2)}_{\frac{i\sqrt{3}}{2}}(\omega l)$

Region II: $-b_0 \le l \le b_0 \Rightarrow \left[-\frac{d^2}{dl^2} + \frac{1}{{b_0}^2}\right] u(l) = \omega^2 u(l) \Rightarrow$ $u_{II}(l) = B_1 e^{ikl} + B_2 e^{-ikl}$

Region III: $l \ge b_0 \Rightarrow \left[-\frac{d^2}{dl^2} + \frac{1}{l^2}\right] u(l) = \omega^2 u(l) \Rightarrow$ $u_{III}(l) = C_1 \sqrt{l} H^{(1)}_{\frac{i\sqrt{3}}{2}}(\omega l)$

Applying the boundary conditions, we could found B_1, B_2 and C_1. The last boundary condition generates the Transcendental Equation (3).

$$\begin{aligned} &2\sqrt{X^2-1}\sin(2\sqrt{X^2-1}) - \cos(2\sqrt{X^2-1})\left[-1+2Xf(-X)\right] + \\ &+\left[1+2Xg(X)\right]\left[\frac{\sin(2\sqrt{X^2-1})}{2\sqrt{X^2-1}}(-1+2Xf(-X)) + \cos(2\sqrt{X^2-1})\right] = 0. \end{aligned} \tag{3}$$

Where $f(-\omega b_0) \equiv \frac{\dot{H}^{(2)}_{\frac{i\sqrt{3}}{2}}(-\omega b_0)}{H^{(2)}_{\frac{i\sqrt{3}}{2}}(-\omega b_0)}$, $g(\omega b_0) \equiv \frac{\dot{H}^{(1)}_{\frac{i\sqrt{3}}{2}}(\omega b_0)}{H^{(1)}_{\frac{i\sqrt{3}}{2}}(\omega b_0)}$ and $X \equiv \omega b_0$.

Conclusions

In conclusion, we could found rather simple forms of density, pressure and tension for the minimal wormhole. Also, the Effective Potential for a massless scalar perturbation resembles the classical Box Potential in Quantum Mechanics, which suggests a scattering problem.

In fact, solving a scattering problem where the incident waves are eliminated, a transcendental equation was found, whose solution are the quasinomal frequencies of the object. In the specific case of this minimal wormhole, the equation is quite complex, requiring, as the next step of our research, a computational method to solve it.

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